

3. VERTICAL STRUCTURE OF THE WIND-DRIVEN CIRCULATION

In lecture 2 we considered a model of the wind-driven circulation with no vertical structure.

However observations show that the strongest flows (and strongest density variations) are concentrated in the upper few hundred meters of the ocean.

In this lecture we will introduce the dynamics that sets the vertical structure of the wind-driven circulation, specifically:

- the surface Ekman layer,
- the role of potential vorticity in the ocean interior.

This lecture will contain no discussion of western boundary currents. Some of the results are therefore of a tentative nature, in that there is an implicit assumption a boundary current solution can be found to close the gyre without feeding back onto its structure.

THE EKMAN LAYER

The direct effect of the wind stress is only felt within the upper 30-100m of the ocean, known as the *Ekman Layer*.

Within the Ekman layer, to leading order:

$$-fv + \frac{1}{\rho_0} \frac{\partial p}{\partial x} = \frac{1}{\rho_0} \frac{\partial \tau^{(x)}}{\partial z}, \quad (3.1)$$

$$fu + \frac{1}{\rho_0} \frac{\partial p}{\partial y} = \frac{1}{\rho_0} \frac{\partial \tau^{(y)}}{\partial z}, \quad (3.2)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (3.3)$$

with $\tau = \tau_s$ at the sea surface and $\tau \rightarrow 0$ at the base of the Ekman layer.

We now decompose the velocity into *geostrophic* and wind-driven *Ekman* components:

$$\mathbf{u} = \mathbf{u}_g + \mathbf{u}_{Ek}, \quad (3.4)$$

such that

$$u_g = -\frac{1}{\rho_0 f} \frac{\partial p}{\partial y}, \quad v_g = \frac{1}{\rho_0 f} \frac{\partial p}{\partial x}, \quad (3.5)$$

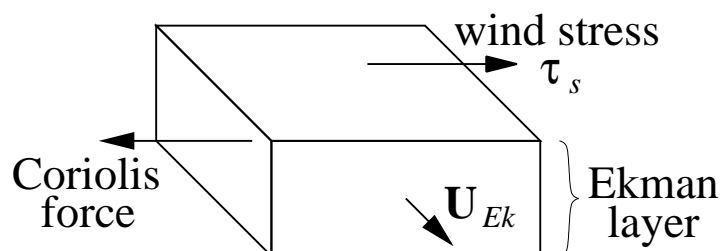
and

$$u_{Ek} = \frac{1}{\rho_0 f} \frac{\partial \tau^{(y)}}{\partial z}, \quad v_{Ek} = -\frac{1}{\rho_0 f} \frac{\partial \tau^{(x)}}{\partial z}. \quad (3.6)$$

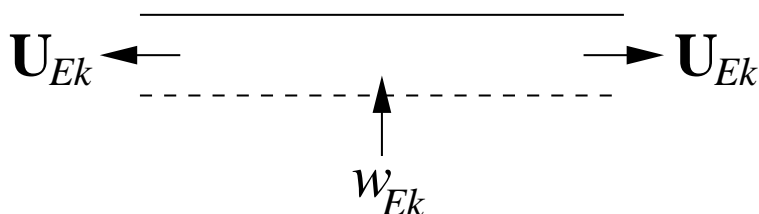
Integrating over the depth of the Ekman layer, the net *Ekman transport* is:

$$U_{Ek} = \frac{\tau_s^{(y)}}{\rho_0 f}, \quad V_{Ek} = -\frac{\tau_s^{(x)}}{\rho_0 f}. \quad (3.7)$$

The Ekman transport is to the right of the wind-stress due to the balance between the wind stress and Coriolis force.



Divergent/convergent Ekman transports \Rightarrow Ekman upwelling/downwelling, w_{Ek} , through the base of the Ekman layer.



Integrating (3.3) over the depth of the Ekman layer gives:

$$w_{Ek} = \frac{\partial}{\partial x} \left(\frac{\tau_s^{(y)}}{\rho_0 f} \right) - \frac{\partial}{\partial y} \left(\frac{\tau_s^{(x)}}{\rho_0 f} \right). \quad (3.8)$$

Generally, $w_{Ek} < 0$ in the subtropical ocean and $w_{Ek} > 0$ in subpolar ocean — see wind-stress data on page 1-6.

Applications:

a. Coastal upwelling

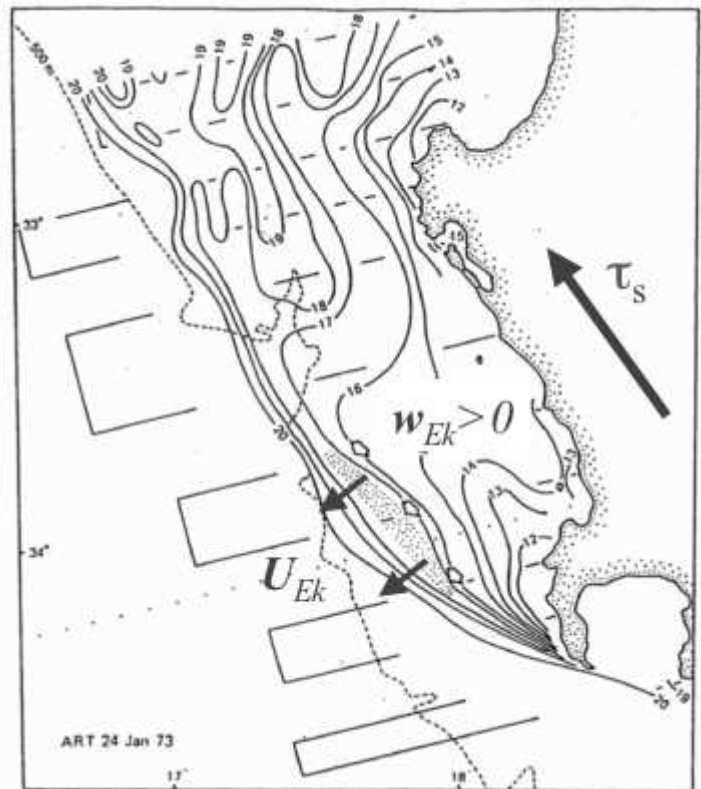
SST off west coast of South Africa (Gill 1982)

Equatorward τ_s

\Rightarrow off-shore \mathbf{U}_{Ek}

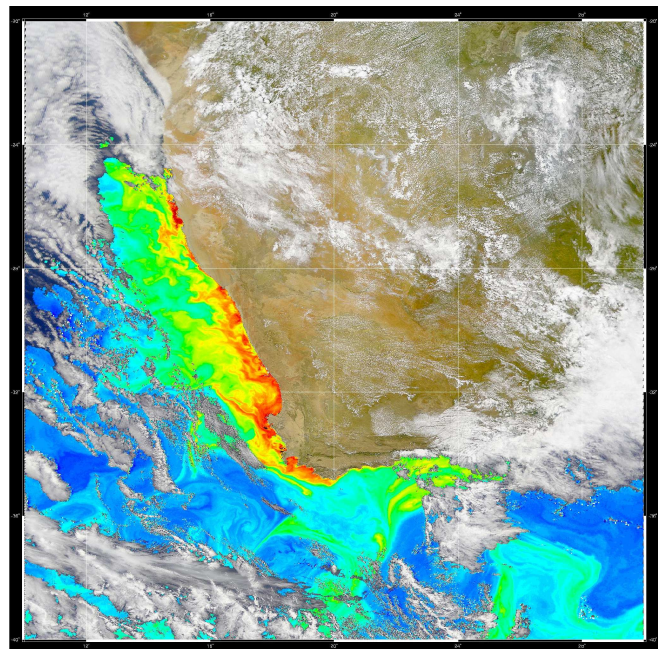
\Rightarrow coastal upwelling

\Rightarrow cold, nutrient-rich water brought to surface.



\Rightarrow many major fisheries found on eastern margins of ocean basins

Surface chlorophyll off west coast of South Africa (seawifs.gsfc.nasa.gov)



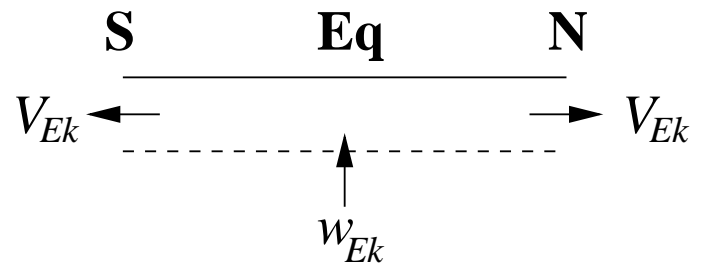
b. Equatorial upwelling

Easterly trade winds

$\Rightarrow V_{Ek} > 0$ north of equator

and $V_{Ek} < 0$ south of equator

\Rightarrow equatorial upwelling.



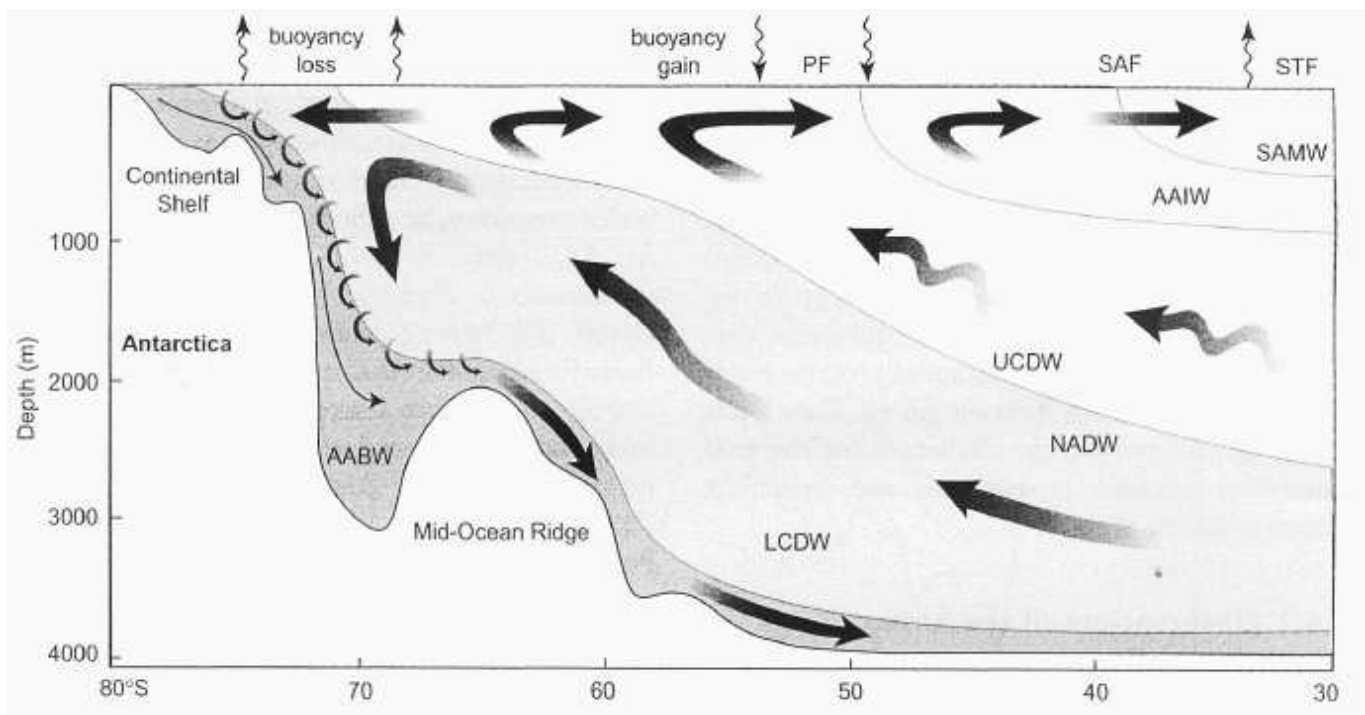
This upwelling is the cause of the *cold pool* in the eastern tropical Pacific (see SST map on page 1-8).

c. Antarctic Circumpolar Current

Westerly winds over the Southern Ocean

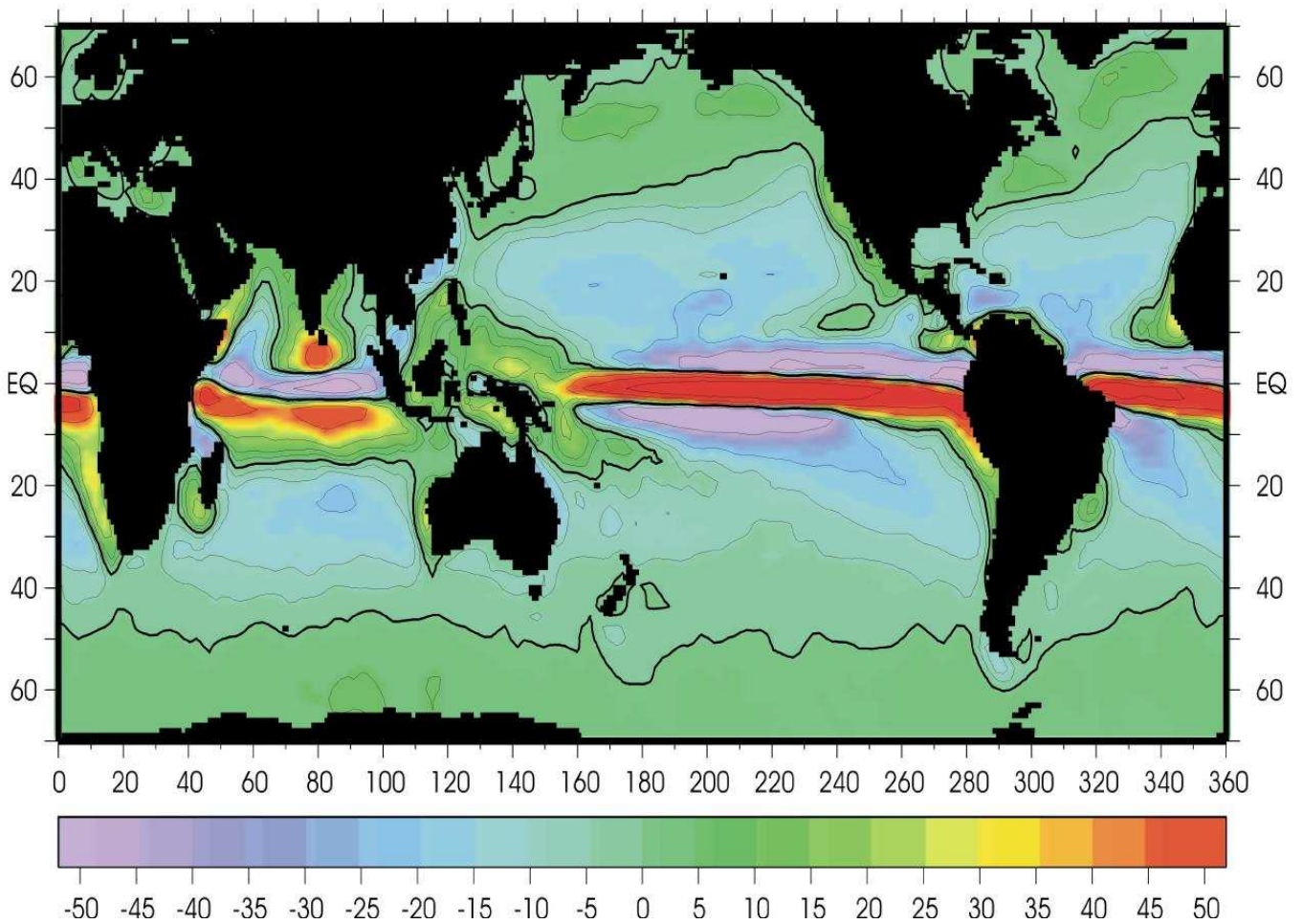
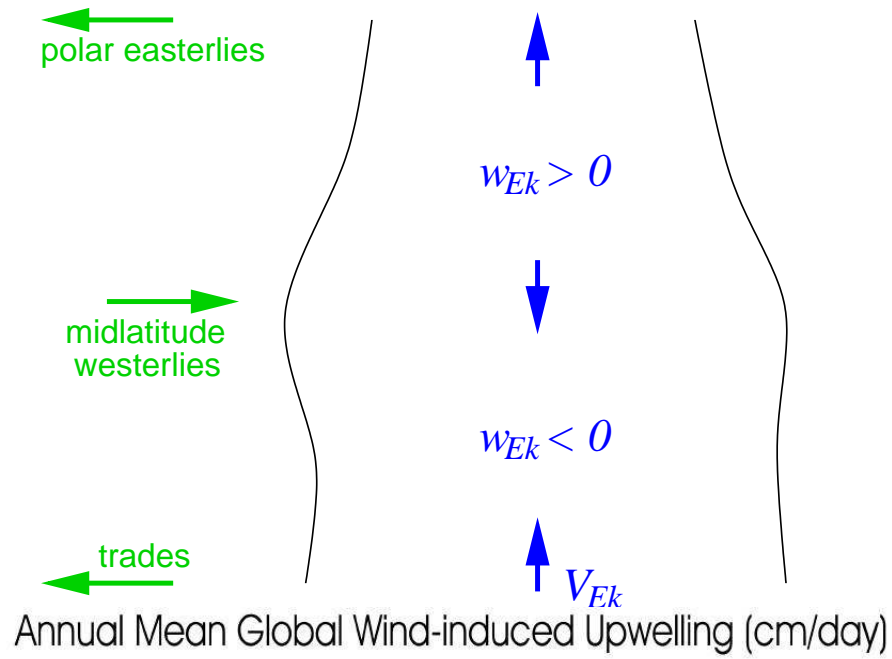
\Rightarrow northward Ekman transport

\Rightarrow ACC can be viewed as a huge coastal upwelling current:



(from Rintoul et al. 2001)

d. Open ocean



(Xie and Hsieh 1995) $1 \text{ cm day}^{-1} \sim 10^{-7} \text{ m s}^{-1}$

SVERDRUP BALANCE

Beneath the Ekman layer, it is straightforward to show that Sverdrup balance carries over to a stratified ocean:

$$\beta \int_{-H}^0 v \, dz = \frac{1}{\rho_0} \left\{ \frac{\partial \tau_s^{(y)}}{\partial x} - \frac{\partial \tau_s^{(x)}}{\partial y} \right\} \quad (3.9)$$

(provided the flow vanishes at depth).

However, more useful for this lecture is a related form of Sverdrup balance for the depth-integrated flow *beneath* the Ekman layer.

Here the flow is in geostrophic balance:

$$u = -\frac{1}{\rho_0 f} \frac{\partial p}{\partial y}, \quad v = \frac{1}{\rho_0 f} \frac{\partial p}{\partial x}. \quad (3.10)$$

Substituting this into the continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (3.11)$$

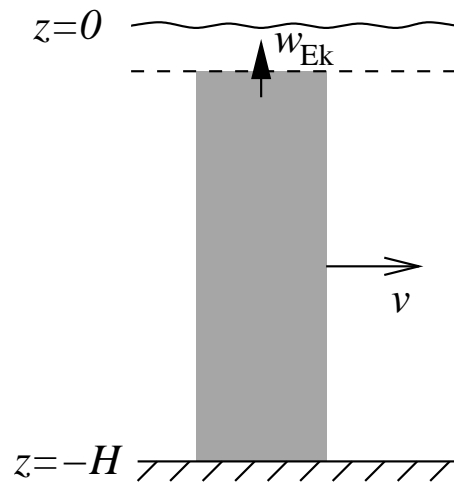
gives the large-scale vorticity balance:

$$\beta v = f \frac{\partial w}{\partial z}. \quad (3.12)$$

Finally integrating from the sea floor ($z = -H$) where $w \approx 0$ to the base of the Ekman layer ($z = z_{Ek}$) where $w = w_{Ek}$ gives:

$$\beta \int_{-H}^{z_{Ek}} v \, dz = f w_{Ek}. \quad (3.13)$$

Physically, if a fluid column is stretched, then it must move polewards to increase its planetary vorticity:



(Q: why not increase its relative vorticity?)

Since β is always positive, the sign of the depth-integrated meridional velocity therefore depends on the sign of f and w_{Ek} :

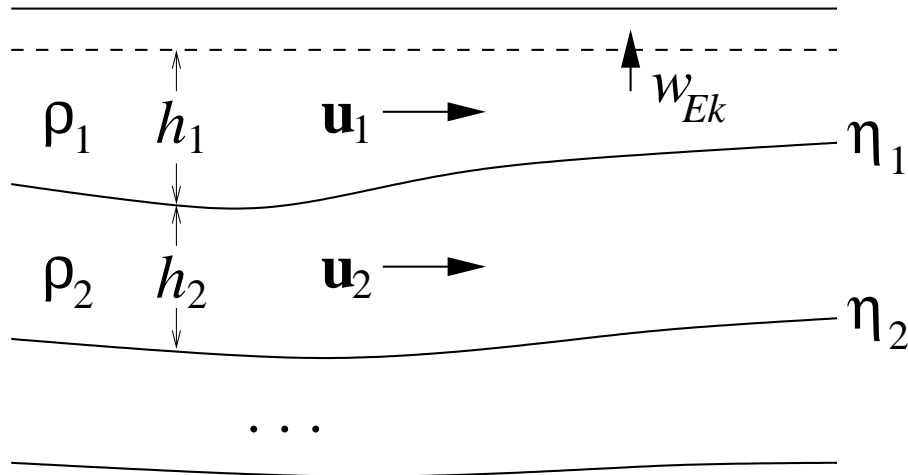
- where $w_{Ek} > 0$, the depth-integrated flow is *poleward*;
- where $w_{Ek} < 0$, the depth-integrated flow is *equatorward*;

However, Sverdrup balance tells us nothing about how the circulation is partitioned over the fluid column.

To solve this problem in a 3-d stratified ocean is extremely challenging \Rightarrow try to use a simpler model.

THE LAYERED MODEL

Approximate the ocean as a series of layers ($n = 1, 2, \dots$), each of constant but different density, ρ_n :



Key dynamical ingredients:

- layered form of Sverdrup balance:

$$\beta \sum v_n h_n = f w_{Ek}. \quad (3.14)$$

- layered form of thermal wind balance:

$$\mathbf{u}_{n+1} = \mathbf{u}_n + \frac{g'_n}{f} \mathbf{k} \times \nabla \eta_n, \quad (3.15)$$

where $g'_n = g(\rho_{n+1} - \rho_n)/\rho_0$.

- conservation of potential vorticity in absence of forcing:

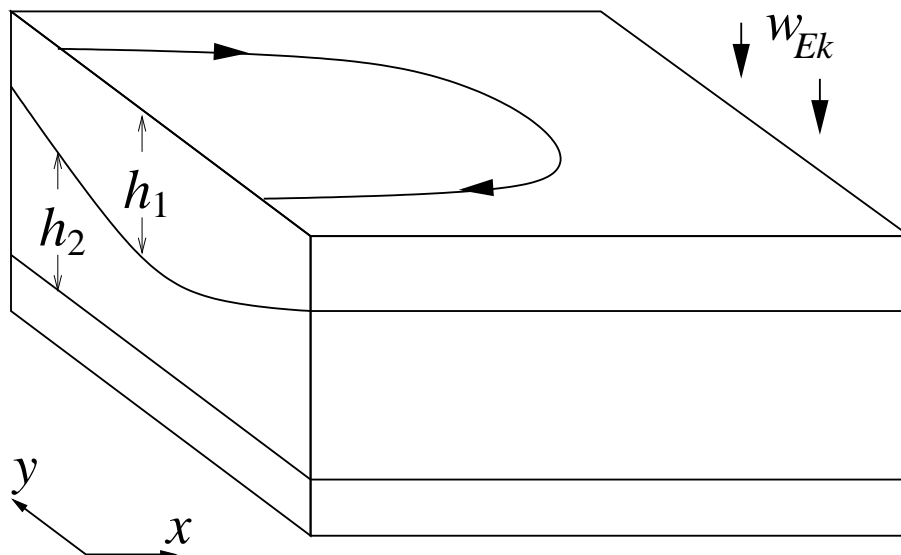
$$\mathbf{u}_n \cdot \nabla q_n = 0, \quad (3.16)$$

where $q_n = f/h_n$ is the large-scale potential vorticity.

RHINES AND YOUNG (1982A, B)

Will omit mathematical details (see Pedlosky 1996).

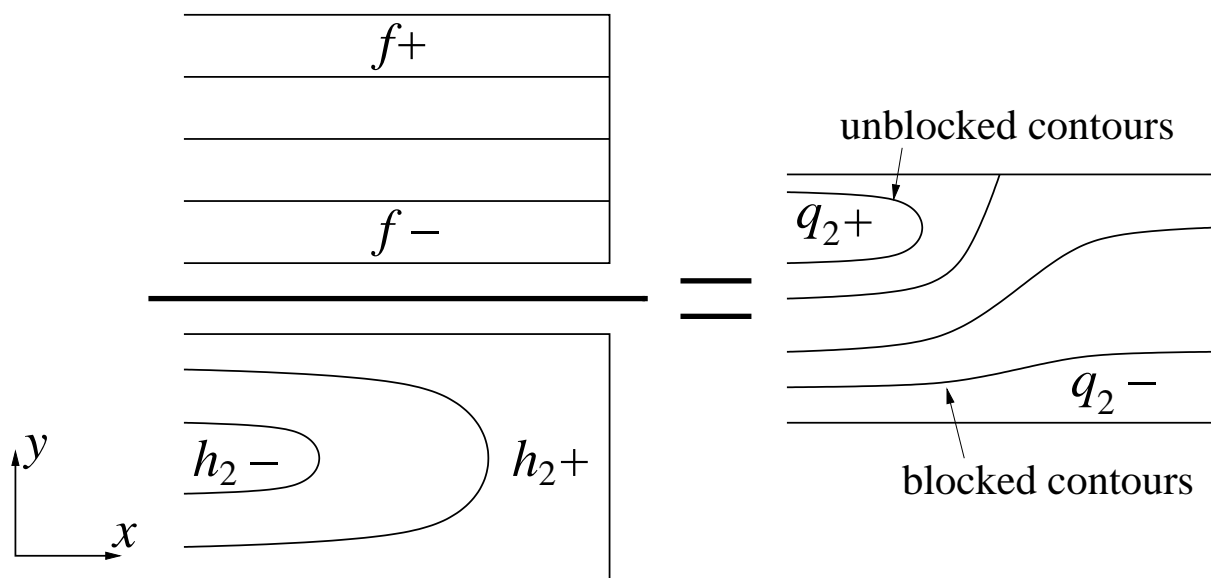
Consider a subtropical gyre, and initially assume flow is confined to layer 1, which is directly exposed to w_{Ek} :



Thermal wind balance

\Rightarrow interface between layers 1 and 2 must deform.

\Rightarrow potential vorticity field in layer 2 modified:



Flow in layer 2 must conserve its potential vorticity.

If we assume that boundary currents can form at the western margins of basins

\Rightarrow flow possible in only the NW corner of layer 2.

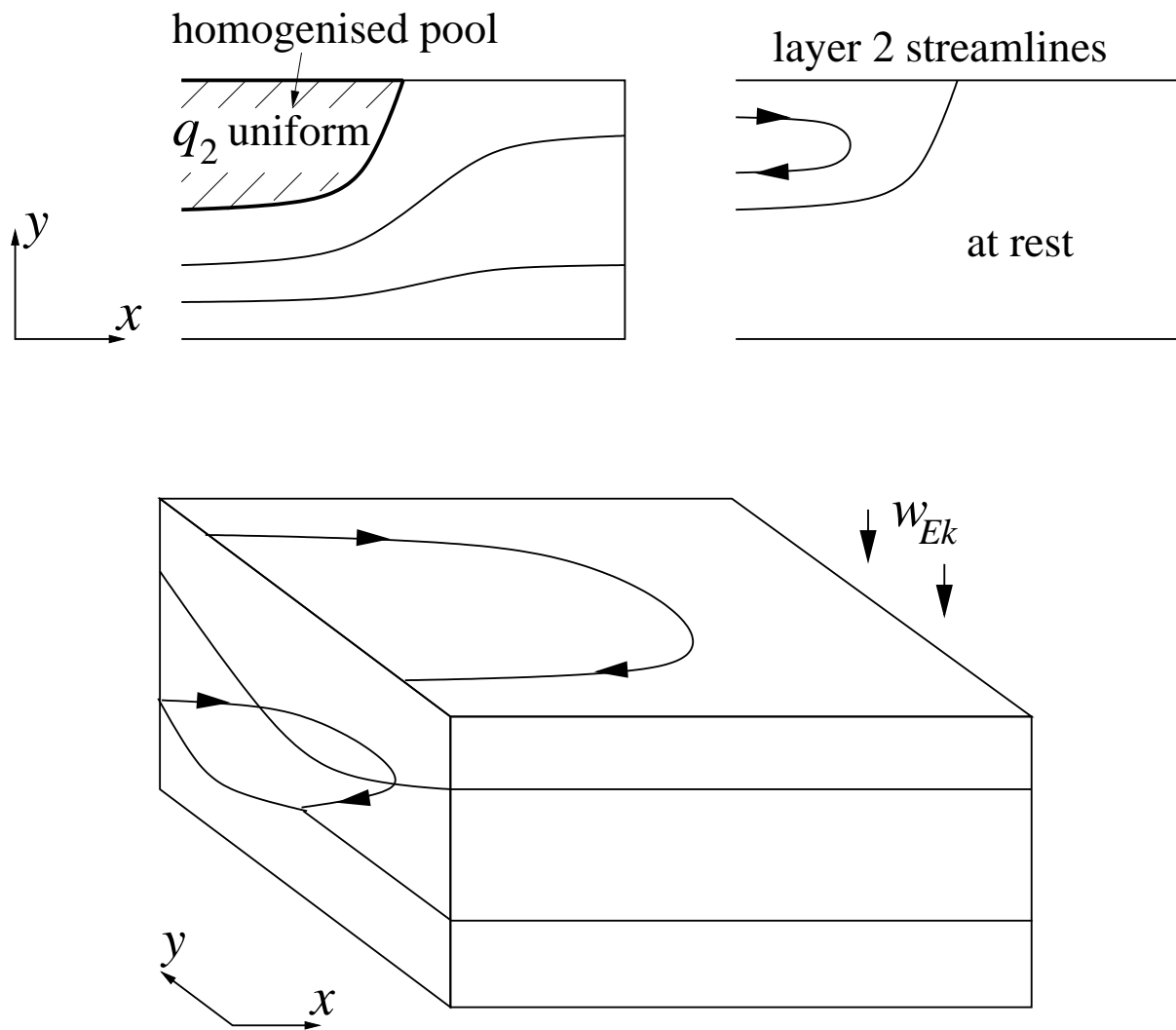
Finally, we need to determine the strength of this flow.

Rhines and Young argued that eddies would homogenize q_2 in this region:



Snapshot of q_2 over the subtropical gyre from an idealised eddy-resolving numerical calculation (Rhines and Young 1982b)

Solution including flow in layer 2:



NB: flow in layer 2

$\Rightarrow q_3$ contours deformed

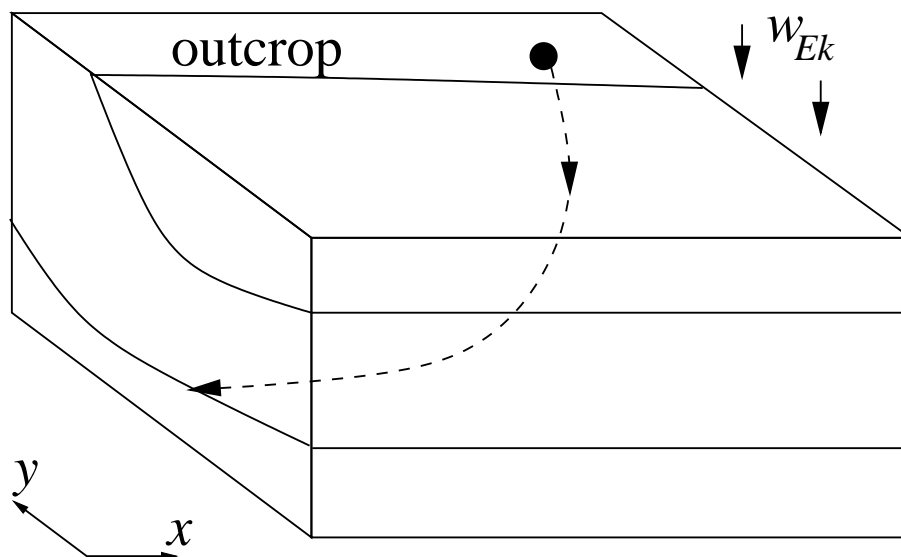
\Rightarrow flow in layer 3? etc

VENTILATION

Surface density increases with increasing latitude
 \Rightarrow density layers will “outcrop” at the sea surface.

There is now the additional possibility of a fluid parcel starting at the sea surface and being *subducted* onto a subsurface layer.

Once shielded from surface forcing, this parcel will conserve its potential vorticity.

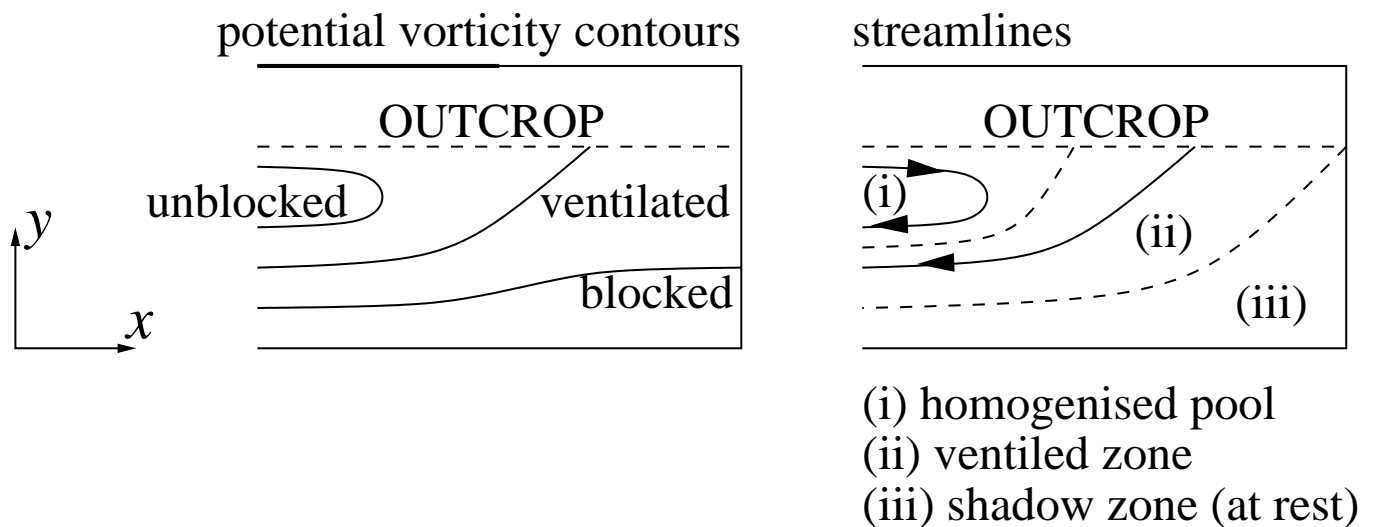


This is the basic idea behind the *ventilated thermocline* model of Luyten, Pedlosky and Stommel (1983) in which surface density variations are mapped onto a vertical stratification through potential vorticity conservation.

There are now 3 types of potential vorticity contours:

- *unblocked contours* that recirculate through the western boundary current (the “homogenised pool”);
- *ventilated contours* that thread down from the sea surface (the “ventilated zone”);
- *blocked contours* that intersect the eastern boundary (the “shadow zone”).

Flow is possible on the first two of these, but not in the shadow zone:

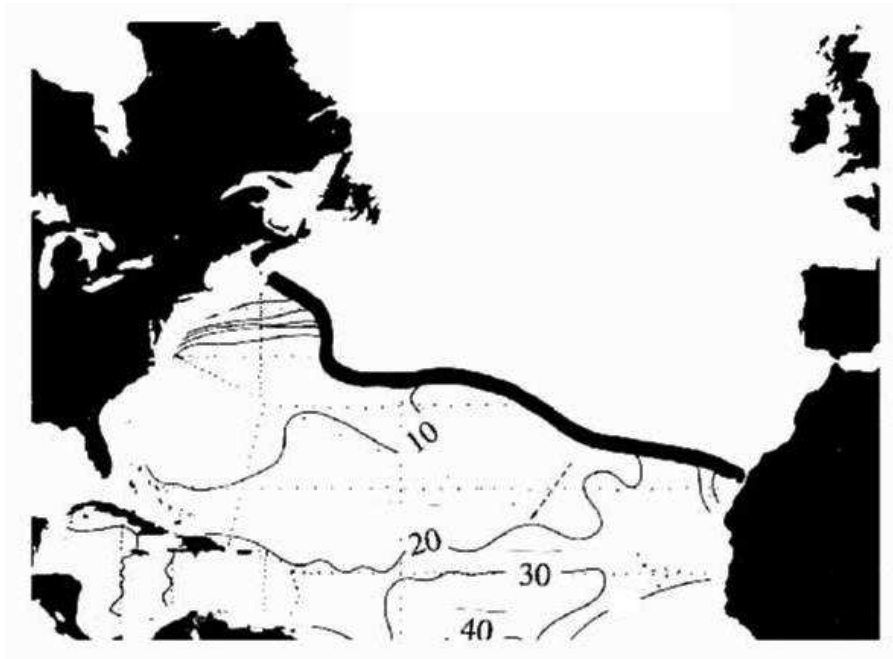


Find that ventilated zone dominates near the surface.

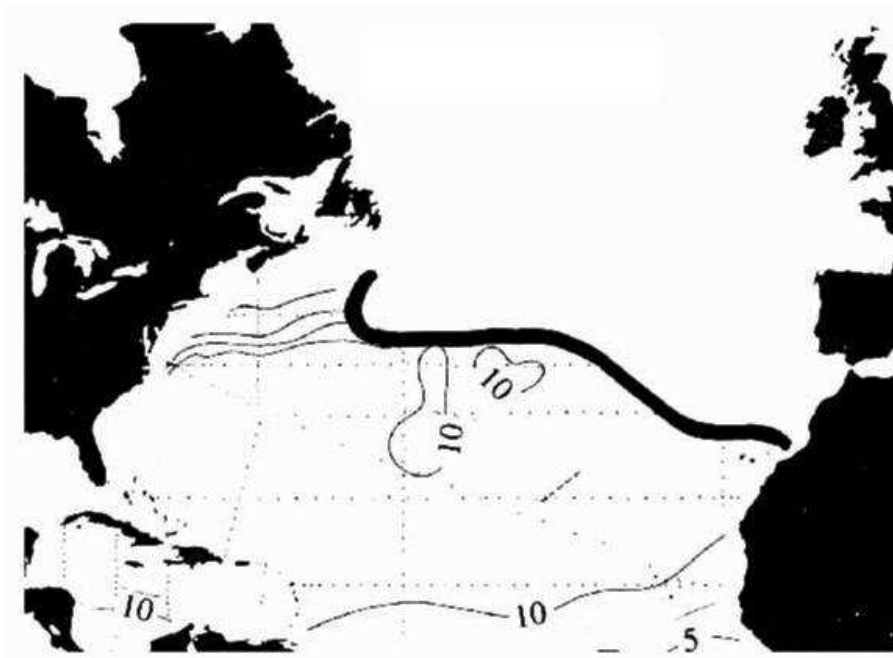
Deeper down the solution resembles that of the Rhines and Young model.

Potential vorticity in the North Atlantic:

$$\sigma_{\theta} = 26.3 - 26.5$$



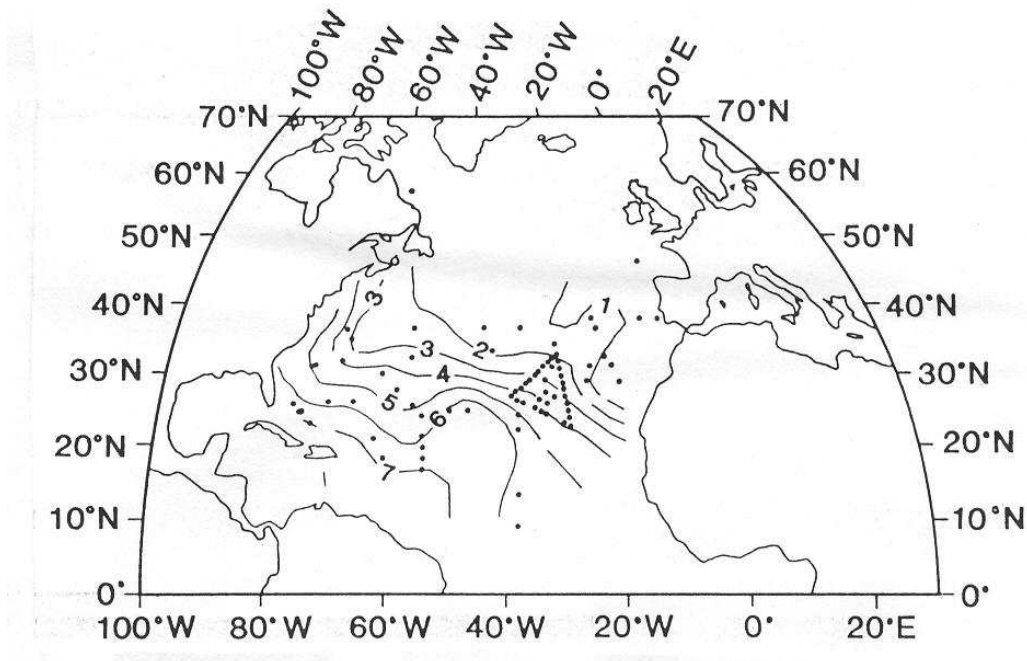
$$\sigma_{\theta} = 26.5 - 27.0$$



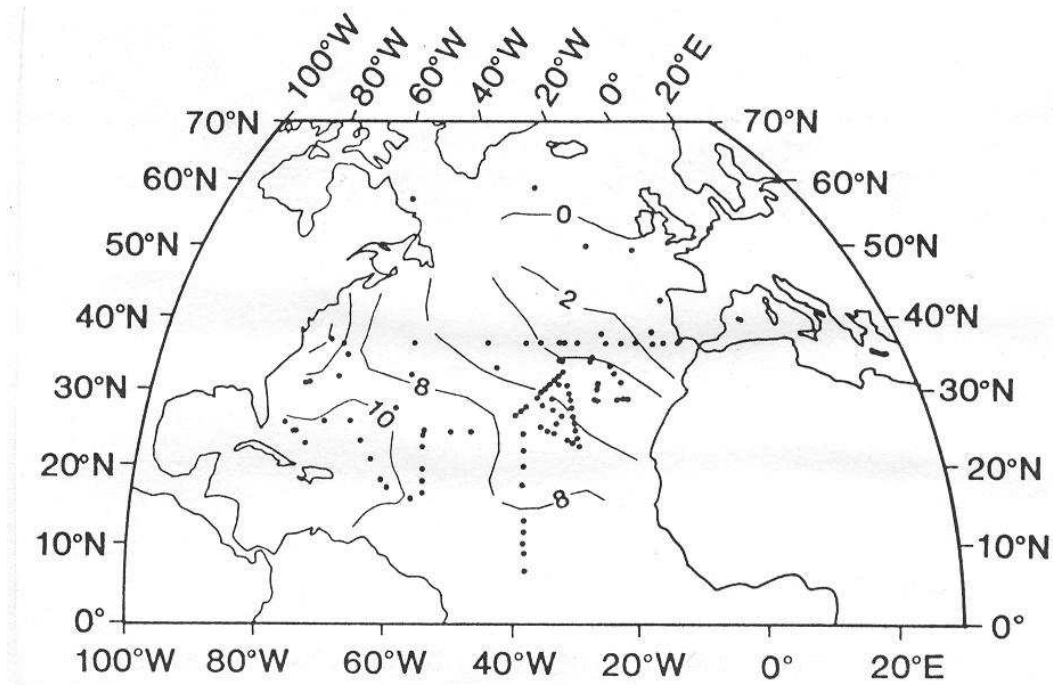
(adapted from McDowell et al. 1982)

“Ventilation age” (from $\text{Tr}/^3\text{He}$ ratio):

$$\sigma_\theta = 26.5$$



$$\sigma_\theta = 26.75$$



(Jenkins 1988)

STOMMEL'S MIXED LAYER DEMON

The properties of the ocean interior match those of the *winter*, rather than annual-mean, surface mixed layer (Iselin 1939). Explained by Stommel (1979).

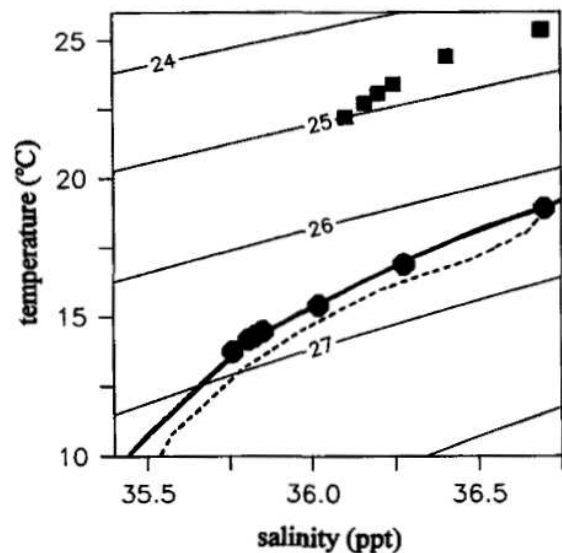
Model calculation:

solid line - 30N, 30W

circles - winter mixed layer

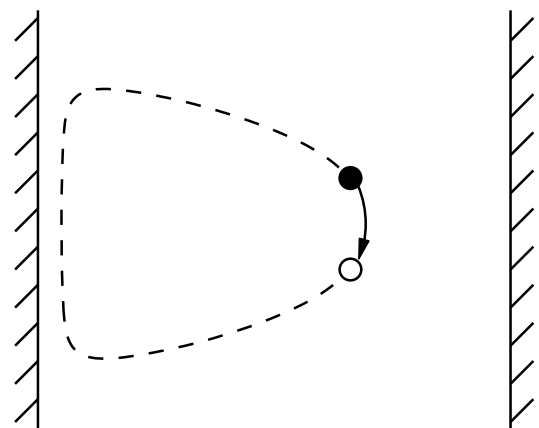
squares - summer mixed layer

(Williams et al. 1995)

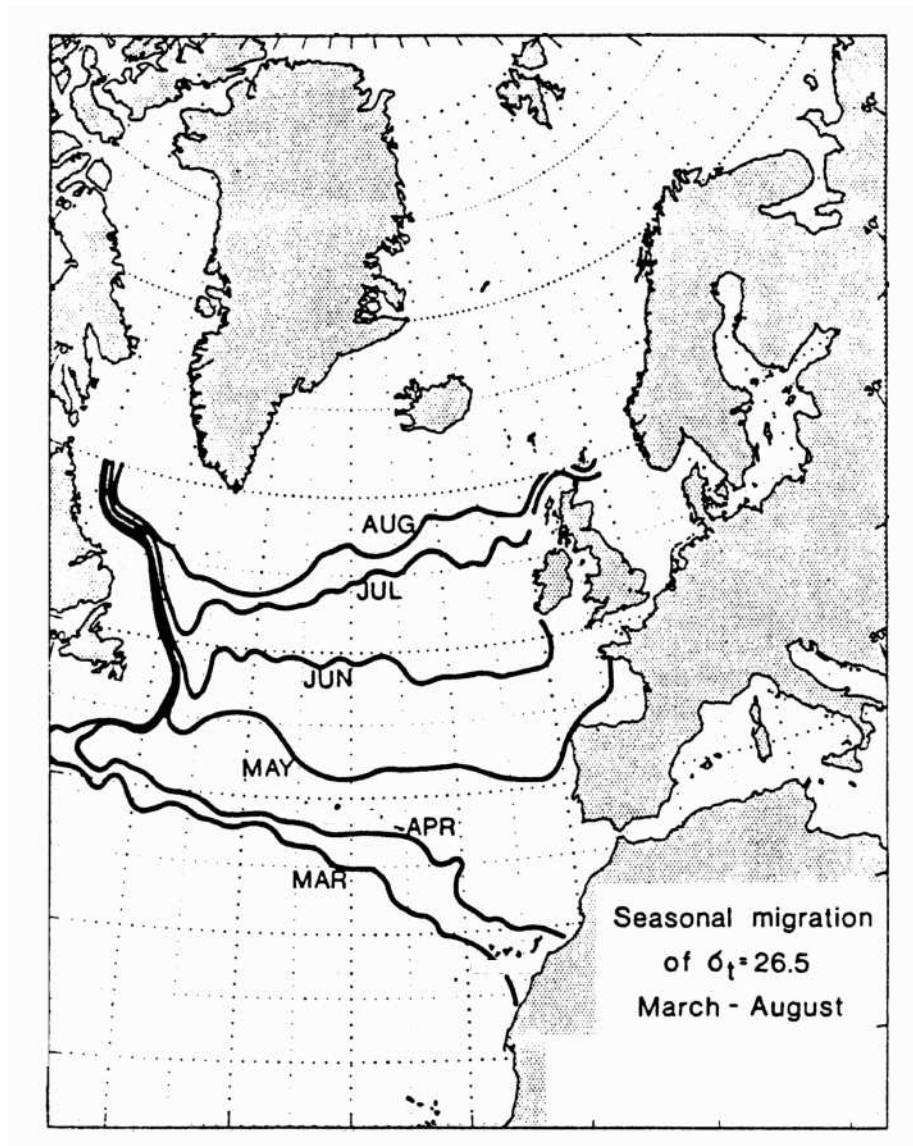


Suppose a fluid parcel in a subtropical gyre is subducted from the surface mixed layer into the ocean interior at a particular time of year. To remain in the ocean interior, the fluid parcel must remain permanently south of the density outcrop.

Over a year, a fluid parcel moves equatorward by a distance $u \Delta t$
 $\sim 10^{-2} \text{ m s}^{-1} \cdot 3 \times 10^7 \text{ s}$
 $= 3 \times 10^5 \text{ m} = 300 \text{ km}.$



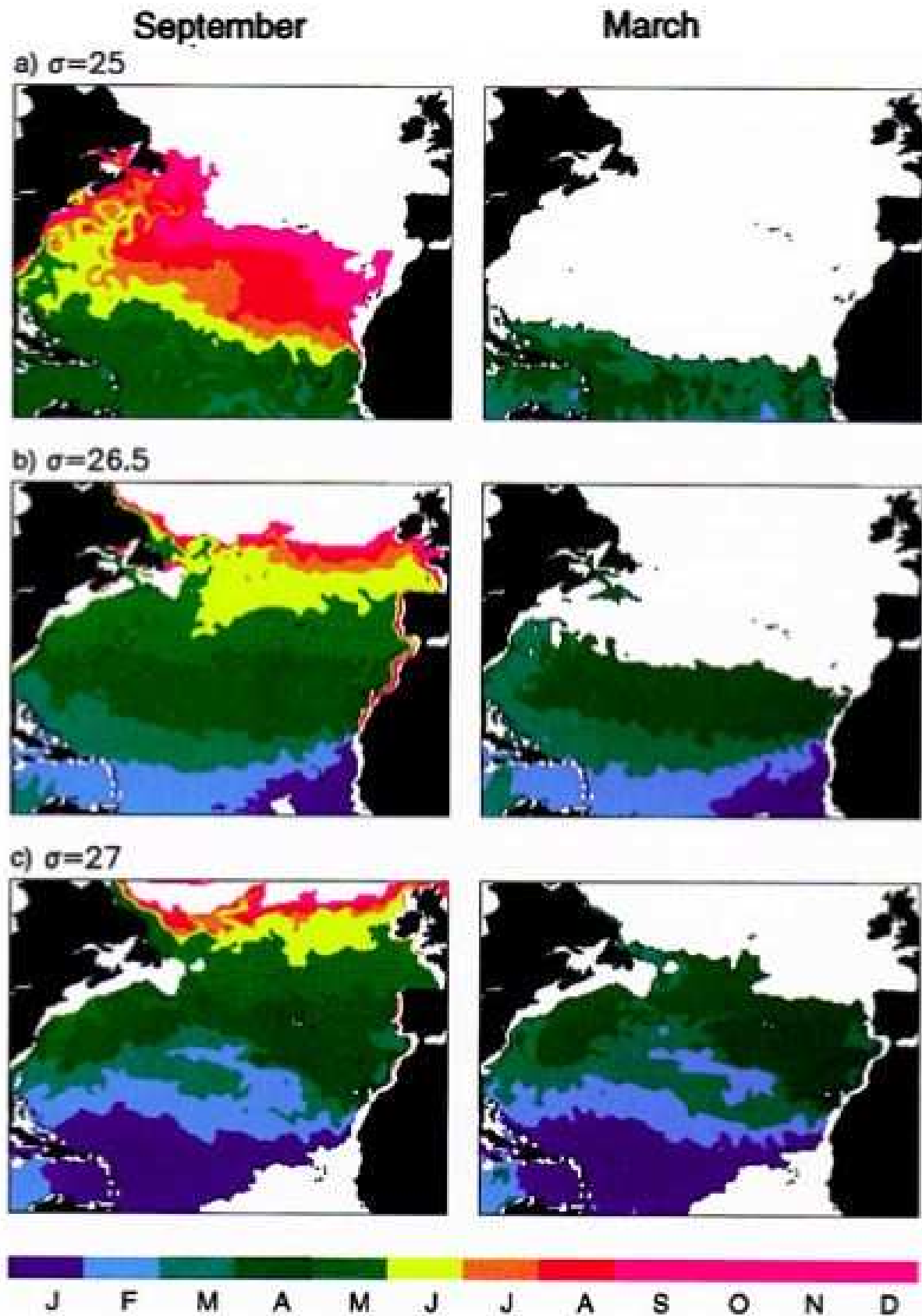
However, surface outcrops migrate by up to 3000 km over the annual cycle:



⇒ most fluid parcels are reentrained into the mixed layer the following winter.

Only fluid parcels subducted in late winter are able to escape the migrating outcrop and remain permanently in the ocean interior.

Williams et al. (1995) injected a *date tracer* into the mixed layer of an eddy-resolving ocean model:



SUMMARY OF MAIN POINTS

- Surface winds drive Ekman transports to the right of the wind stress in the Northern Hemisphere and to the left of the wind-stress in the Southern Hemisphere.
- Divergence/convergence in the lateral Ekman transports \Rightarrow Ekman upwelling/downwelling.
- The vertical structure of the wind-driven circulation is controlled by the geometry of the potential vorticity field.
- The properties of the ocean interior match those of the winter mixed layer.

REFERENCES FOR LECTURE 3

General reading

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Specific references

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